Quiz 3 review key

Stat 301

Summer 2019

(1) Find the probability of the following z-scores:
  (a) $P(Z < 1.89) = 0.970621$
  (b) $P(Z > -0.5) = 1 - P(Z < -.5) = 0.6914625$
  (c) Find the z-score that represents the top 9% $z_{top9\%} = z_{bottom91\%} = 1.340755$
  (d) $P(-1 < Z < 0.87) = P(Z < 0.87) - P(Z < -1) = 0.8078498 - 0.1586553 = 0.6491945$

(2) Given that $X \sim N(33, 3.15)$, find the following probabilities:
  (a) $P(X > 41)$
  $= P(Z > \frac{41-33}{3.15}) = P(Z > 2.54) = 1 - P(Z < 2.54) = 0.0055477$
  (b) $P(X < 27)$
  $= P(Z < \frac{27-33}{3.15}) = P(Z < -1.90) = 0.0284055$
  (c) $P(25 < X < 40)$
  $= P(\frac{25-33}{3.15} < Z < \frac{40-33}{3.15}) = P(-2.54 < Z < 2.22) = P(Z < 2.22) - P(Z < -2.54) = 0.9813182$
  (d) Find the value of X (not just z) that represents the bottom 25% of the distribution
  $z_{.25} = -0.6744898$ and solve for x
  $z = \frac{x-\mu}{\sigma} \Rightarrow x = z\sigma + \mu$
  $x = 30.8753573$

(3) The reaction time (in seconds) to a certain stimulus is a continuous random variable with pdf:

$$f(x) = \begin{cases} \frac{3}{2x^2} & 1 \le x \le 3 \\ 0 & otherwise \end{cases}$$

  (a) Find $F_X(x)$ (cumulative density function)
  $F_X(x) = \int_1^x \frac{3}{2x^2}\ dx = \frac{3}{2}\int_1^x \frac{1}{x^3}\ dx = \left(\frac{3}{2}\right)\left(\frac{y^{-1}}{-1}\right)|_1^x = \left(-\frac{3}{2y}\right)|_1^x = \left(-\frac{3}{2x}\right) - \left(-\frac{3}{2}\right) = \frac{3}{2} - \frac{3}{2x}$ so

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{3}{2} - \frac{3}{2x} & 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$$

  (b) What is the probability that reaction time is at most 2.5 seconds?
  $P(X \le 2.5) = F_X(2.5) = \frac{3}{2} - \frac{3}{2(2.5)} = 0.9$
  (c) What is the probability that reaction time is between 1.5 and 2.5 seconds?
  $P(1.5 < X < 2.5) = F_X(2.5) - F_X(1.5) = \left(\frac{3}{2} - \frac{3}{2(2.5)}\right) - \left(\frac{3}{2} - \frac{3}{2(1.5)}\right) = 0.4$
  (d) Compute the expected reaction time
  $EX = \int_1^3 x\left(\frac{3}{2x^2}\right)\ dx = \frac{3}{2}\int_1^3 x^{-1}\ dx = \frac{3}{2}ln(|x|)|_1^3 = \frac{3}{2}(ln3 - ln1) = 1.6479184$
  (e) Compute the variance and standard deviation
  $VX = E(X^2) - (EX)^2$ with $E(X^2) = \int_1^3 x^2\left(\frac{3}{2x^2}\right)\ dx = \frac{3x}{2}|_1^3 = \frac{3(3)-3(1)}{2} = 3$ and $VX = 0.2843648$
  and $SDX = \sqrt{VX} = 0.5332587$

(4) A subway train on the Red Line arrives every eight minutes during rush hour uniformly. Of interest is the length of time a commuter must wait for a train to arrive during rush hour.
  (a) State the distribution and its parameter(s)
  $X \sim unif(0, 8)$
  (b) Find $F_X(x)$ (cumulative density function)
  $F_X(x) = \frac{X-A}{B-A} = \frac{X-0}{8-0} = \frac{x}{8}$

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{x}{8} & 0 \le x \le 8 \\ 1 & x > 8 \end{cases}$$

(c) Find the probability that the commuter waits less than one minute
$P(X < 1) = F_X(1) = \frac{1}{8}$

(d) Find the probability that the commuter waits between three and four minutes
$P(3 < X < 4) = F_X(4) - F_X(3) = \frac{4}{8} - \frac{3}{8} = \frac{1}{8}$

(e) Find the probability that the commuter waits more than 5 minutes
$P(X > 5) = 1 - P(X < 5) = 1 - F_X(5) = 1 - \frac{5}{8} = \frac{3}{8}$

(f) Compute the expected waiting time
$EX = \frac{B+A}{2} = \frac{8+0}{2} = 4$

(g) Compute the variance and standard deviation
$VX = \frac{(B-A)^2}{12} = \frac{(8-0)^2}{12} = \frac{64}{12} = 5.3333333$, $SDX = \sqrt{VX} = 2.3094011$

(5) During the years 2003-2018, a total of 31 earthquakes of magnitude greater than 6.5 have occurred in Papua New Guinea. Assume that the time spent waiting between earthquakes is exponential.

(a) What is the decay rate λ? State the distribution with shorthand notation.
There are 15 years and over those 15 years, there have been 31 earthquakes. If we are interested in the average number of earthquakes per year, then $\mu = \frac{31}{15} = 2.0666667$. If we are interested in average time between quakes, then $\mu = \frac{15}{31} = 0.483871$ and then $\lambda = \frac{1}{\mu} = \frac{1}{15/31} = \frac{31}{15} = 2.0666667$

(b) What is the probability that the next earthquake occurs within the next three months?
$T \sim exp(2.0666667)$; $P(T < \frac{3}{12}) = 1 - e^{-(31/15)(3/12)} = 0.4034944$

(c) Given that six months has passed without an earthquake in Papua New Guinea, what is the probability that the next three months will be free of earthquakes?
$P(T > \frac{9}{12} | T > \frac{6}{12}) = P(T > \frac{3}{12}) = 1 - P(T < \frac{3}{12}) = 1 - (1 - 0.4034944) = 0.5965056$

(d) Calculate EX, VX, SDX
$EX = \frac{1}{\lambda} = \frac{1}{2.067} = 0.483871$, $VX = \frac{1}{\lambda^2} = \frac{1}{2.067^2} = 0.2341311$, $SDX = \sqrt{VX} = 0.483871$